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Dual Descriptions of Supersymmetry Breaking

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Dynamical supersymmetry breaking is considered in models which admit descriptions in terms of electric, confined, or magnetic degrees of freedom in various limits. In this way, a variety of seemingly different theories which break supersymmetry are actually inter-related by confinement or duality. Specific examples are given in which there are two dual descriptions of the supersymmetry breaking ground state.

1. Introduction

Recent advances have shown that supersymmetric gauge theories can often appear in the infrared as other theories, with different gauge groups and matter content [1]. This is the phenomenon of electric-magnetic duality generalized to non-Abelian theories. One possibility is that two or more gauge theories, which differ in the ultraviolet, can flow in the infrared to the same interacting fixed point of the renormalization group. The fixed point theory is in an interacting non-Abelian Coulomb phase which can be described in terms of any of the "dual" ultraviolet theories. Another possibility is that an ultraviolet theory can flow to another gauge theory which is not asymptotically free. The infrared theory is in a free magnetic phase, with weakly coupled magnetic quarks and gluons. Another possibility is that the ultraviolet gauge theory flows to a Wess-Zumino model without gauge interactions [2]. The infrared theory is described in terms of confined fields with tree level interactions. A review of recent work in supersymmetric gauge theories and a list of references is given in [3]. In this paper we study chiral gauge theories in which dynamical supersymmetry breaking admits descriptions in terms of electric, magnetic, or confined degrees of freedom in various limits. The mechanism by which supersymmetry is broken in these theories is a dynamically generated superpotential.

Non-perturbative gauge dynamics can spontaneously break supersymmetry, a phenomenon which can be useful for constructing models with natural hierarchies of scales. It is of interest to see how dynamical supersymmetry breaking is compatible with duality. In one sense, the fact that there are dual descriptions of supersymmetry breaking is actually standard, though perhaps somewhat trivial. Any theory which breaks supersymmetry dynamically with a mass gap is dual in the far infrared to any trivial theory with the same (discrete) vacuum structure. Alternately, if there are gapless excitations in the ground state, such as a $U(1)_R$ axion or massless fermions required for anomaly cancelation, the theory is dual in the far infrared to a non-supersymmetric non-linear sigma model. Here we will explore slightly less trivial notions of dual and confined descriptions, relating supersymmetry breaking in gauge theories to that in other gauge theories. In order to make use of supersymmetric dualities and confinement we consider models with (at least) two well separated dynamical scales, $\Lambda_H \gg \Lambda_L$, with supersymmetry broken at or below the scale Λ_L. In this way there is region of momenta below Λ_H, but above the supersymmetry breaking scale, in which supersymmetry is realized linearly. The effective action is manifestly supersymmetric in this region, and the powerful constraints of supersymmetry may

be implemented.

Below the scale Λ_H, the theories considered here flow towards an interacting fixed point, free magnetic, or confined phase. At the scale Λ_L additional strong dynamics break supersymmetry in the effective low energy theory. In the case of a non-Abelian Coulomb phase, the theory never reaches the fixed point at the origin of moduli space since supersymmetry is broken in the ground state. Duality is exact only at the fixed point, but should apply to the light degrees of freedom in the neighborhood of the fixed point. There can therefore be simultaneous dual descriptions of the supersymmetry breaking in the ground state. In the case of a free magnetic or confined phase there is only one weakly coupled description of the theory below the scale Λ_H. The magnetic or confined description can however be continuously connected to an electric description of the supersymmetry breaking by adjusting the parameters of the theory so that $\Lambda_H \rightarrow 0$ holding Λ_L fixed.

Applications of duality and confinement to supersymmetry breaking are useful for a number of reasons [4]. As a function of the ultraviolet parameters of a theory, the relevant degrees of freedom in the ground state are in some instances confined or magnetic fields with a different gauge group, rather than the underlying electric fields and ultraviolet gauge group. Duality or confinement is therefore required in order to give a proper description of the supersymmetry breaking in these circumstances. In addition, since duality or confinement in general gives a different low energy description of the supersymmetry breaking, it acts as a generator for other models of supersymmetry breaking in which the magnetic or confined fields are re-interpreted as electric fields in the ultraviolet. Finally, it is possible that theories which break supersymmetry by the "classic" mechanism of a dynamically generated superpotential over a classical moduli space are related by duality or confinement to models which break supersymmetry by another mechanism. This technique was employed in Ref. [5] to illustrate a model which breaks supersymmetry in one limit by a dynamically generated superpotential in one gauge group, and in another limit by the quantum deformation of the moduli space due to another gauge group.

In the next section we illustrate confinement and duality in a class of simple renormalizable chiral models which break supersymmetry based on the gauge group $SU(N)\times SU(2)$. These models are related by confinement or duality to the $SU(N)$ models of Affleck, Dine, and Seiberg [6]. Some of the models are near a non-Abelian Coulomb phase and admit simultaneous dual descriptions of the supersymmetry breaking ground state. Others are in a free magnetic, free electric, or confined phase and only admit one weakly coupled de-

scription of the ground state. For these models it is possible to check the scaling of physical quantities such as the vacuum energy, and verify explicitly that the weakly coupled electric and magnetic or confined descriptions do not have overlapping regions of applicability. All these models break supersymmetry by a dynamically generated superpotential in the electric, confined, and magnetic descriptions, and relate generalizations of known models. In section four we discuss generalizations, and comment on connections between models of supersymmetry breaking. In an Appendix we outline non-renormalizable chiral models based on the gauge group $SU(N) \times SP(M)$ which generalize the $SU(N) \times SU(2)$ models.

2. Supersymmetry Breaking by a Dynamical Superpotential

Perhaps the simplest renormalizable chiral model of dynamical supersymmetry breaking with two gauge groups is the $SU(3) \times SU(2)$ model of Affleck, Dine, and Seiberg [6]. In Ref. [5] we showed that in a limit where the $SU(2)$ dynamics dominates, this model breaks supersymmetry by the quantum deformation of the classical moduli space. In this section we consider $SU(N) \times SU(2)$ generalizations of this model, and show that in a limit where the $SU(2)$ dynamics dominates, the theories are either confined, near a non-Abelian Coulomb phase, or free magnetic phase, with gauge group $SU(N) \times SP(\frac{1}{2}(N-5))$ and different matter representations. In all these descriptions, supersymmetry is broken by the dynamically generated $SU(N)$ superpotential which lifts the moduli space.

The matter content of the models is

$$\begin{array}{cc} & SU(N) \times SU(2) \\ & \\ P & (\Box, \Box) \\ L & (1, \Box) \\ \overline{U} & (\overline{\Box}, 1) \\ \overline{D} & (\overline{\Box}, 1) \end{array} \tag{2.1}$$

with N odd. Classically there is a moduli space of vacua parameterized by the invariants $Z = P^2\overline{U}\overline{D}$, $X_1 = PL\overline{D}$, and $X_2 = PL\overline{U}$, with the gauge group generically broken to $SU(N-2) \subset SU(N)$. There is another gauge invariant, $Y = P^N L$, which vanishes classically by Bose statistics of the underlying fields. At tree level there is a single renormalizable coupling which can be added to the superpotential,

$$W_{tree} = \lambda X_1. \tag{2.2}$$

This superpotential leaves invariant non-anomalous $U(1)_R$ and $U(1)$ flavor symmetries, and completely lifts the classical moduli space. Classically there is a supersymmetric ground state at the origin.

Quantum mechanically, the non-perturbative $SU(N)$ dynamics lifts the classical supersymmetric ground state $Z = X_i = 0$, and both the $U(1)_R$ and supersymmetry are spontaneously broken. It follows from the $U(1)_R$ and $U(1)$ flavor symmetries that the $SU(2)$ dynamics do not lift the moduli space. As discussed in the following subsections, even though only the $SU(N)$ dynamics lifts the moduli space, the low energy description of supersymmetry breaking in the ground state depends on the relative importance of the $SU(N)$ and $SU(2)$ non-perturbative dynamics. This is because extra confined or magnetic degrees of freedom associated with the $SU(2)$ can become light near the origin.

2.1. Electric Description

If the $SU(2)$ is weakly gauged in the ground state, any concomitant non-perturbative effects may be ignored, and the $SU(2)$ may be treated classically. The exact superpotential over the classical moduli space is then given by

$$W = (N-2)\left(\frac{\Lambda_N^{3N-2}}{Z}\right)^{1/(N-2)} + \lambda X_1. \tag{2.3}$$

The dynamical part of this superpotential is due to gaugino condensation in the unbroken $SU(N-2) \subset SU(N)$. For $\lambda \ll 1$, the vacuum expectation values in the ground state are close to the classical moduli space, and large compared to Λ_N. In this weak coupling limit, the relevant degrees of freedom in the ground state are just the classical moduli, which parameterize the projection of the elementary electric fields onto the classical moduli space. Parametrically, for $\lambda \ll 1$, the field expectation values and vacuum energy scale as $\phi \sim \lambda^{-(N-2)/(3N-2)}\Lambda_N$ and $V \sim |\lambda^{2(N+2)/(3N-2)}\Lambda_N^4|$. In order for this approximation to be valid, the non-perturbative $SU(2)$ dynamics must be unimportant at the scale of the expectation values. For $N \leq 9$, the $SU(2)$ is asymptotically free and the requirement of weak coupling amounts to $\Lambda_N \gg \lambda^{(N-2)/(3N-2)}\Lambda_2$, with Λ_2 the scale of the $SU(2)$. For $N \geq 11$, the $SU(2)$ is not asymptotically free, and weak coupling in the ground state requires $\Lambda_N \ll \lambda^{(N-2)/(3N-2)}\Lambda_2$. In these limits, the $SU(2)$ acts as a spectator in the non-perturbative $SU(N)$ dynamics which breaks supersymmetry. Its only role is to provide a classical gauge potential which lifts certain directions in field space. In the following subsections we consider the limits in which the $SU(2)$ dynamics is important.

2.2. Confined Description

If the $SU(2)$ is strongly coupled in the ground state, its non-perturbative dynamics can not be ignored. For $N = 5$, in the limit $\Lambda_2 \gg \Lambda_5$, the $SU(5)$ is weakly gauged at the scale Λ_2 and may be treated as a weakly gauged subgroup of an $SU(6)_F$ flavor symmetry under which P and L are not distinguished. The $SU(2)$ theory therefore has three flavors (six $\Box$) and confines as in Ref. [2]. The resulting confined theory is $SU(5)$ with matter given by $\widehat{F} = F/\Lambda_2 = PL/\Lambda_2 \in \Box$ of $SU(5)$, and $\widehat{A} = A/\Lambda_2 = P^2/\Lambda_2 \in \begin{smallmatrix}\Box\\\Box\end{smallmatrix}$ of $SU(5)$ (throughout hatted fields represent canonically normalized confined degrees of freedom). For expectation values much less than Λ_2 these fields, along with $\bar{U}$ and $\bar{D}$, make up the canonically normalized degrees of freedom, as evidenced by the t' Hooft anomaly matching conditions [2]. In the absence of the electric tree level superpotential (2.2), the moduli space of the low energy confined theory is parameterized by $X_1 = F\overline{D}$, $X_2 = F\overline{U}$, $Z = A\overline{U}\overline{D}$, and $Y = A^2F$, subject to the confining superpotential $W_{conf} = -Y/\Lambda_2^3$. This superpotential ensures that the moduli space of the confined theory agrees with that of the high energy electric theory [2]. In particular, just as in the electric theory, the gauge group is generically broken to $SU(3) \subset SU(5)$ on the moduli space of the theory with $SU(2)$ confined.

The scale $\widehat{\Lambda}_5$ of the low energy theory is related to Λ_5 by the matching relation $\widehat{\Lambda}_5^{12} = \Lambda_5^{13}/\Lambda_2$ at the scale Λ_2. The exact superpotential of the low energy theory is then

$$W = 2\left(\frac{\Lambda_5^{13}\Lambda_2^3}{YZ}\right)^{1/2} - \frac{Y}{\Lambda_2^3} + \lambda X_1. \tag{2.4}$$

The first term is generated by gaugino condensation in an unbroken $SU(2) \subset SU(3) \subset SU(5)$, as can be verified by using the matching condition and substituting the invariants of the low energy theory. The second term is the confining superpotential, $W_{conf} = -\widehat{A}^2\widehat{F}$, which is a Yukawa coupling in the low energy theory. The final term is the confined operator corresponding to the electric tree level superpotential (2.2), and gives a Dirac mass $m = \lambda\Lambda_2$ to the pair $\widehat{F}$ and $\overline{D}$ in the effective theory.

For $\lambda\Lambda_2 \gg \widehat{\Lambda}_5$, the Dirac pair $\widehat{F}\overline{D}$ may be integrated out of the effective theory. The remaining matter fields are $\overline{U} \in \Box$ and $\widehat{A} \in \begin{smallmatrix}\Box\\\Box\end{smallmatrix}$ of $SU(5)$. This is the original (non-calculable) model of dynamical supersymmetry breaking given by Affleck, Dine, and Seiberg [7]. This theory has a $U(1)_R$ symmetry and a classical supersymmetric ground state at the origin. This is is presumably lifted by the $SU(5)$ non-perturbative effects, with supersymmetry

broken, as argued in [7,8]. Although this theory does not have a weak coupling limit, the expectation values in the ground state are presumably $\phi \sim \widehat{\Lambda}_5$ and $V \sim \widehat{\Lambda}_5^4$.

For $\lambda\Lambda_2 \ll \widehat{\Lambda}_5$ the Dirac pair $\widehat{F}\overline{D}$ are light compared to the dynamical scale and may not be integrated out. The low energy theory then amounts to the above Affleck, Dine, Seiberg theory with an extra flavor, which was studied (as an electric theory) in Ref. [9]. In the present context, because of the large confining Yukawa coupling $W_{conf} = -\widehat{A}^2\widehat{F}$ in (2.4), the Y modulus may be integrated out by applying its equation of motion, $Y = -(\Lambda_5^{13}\Lambda_2^9/Z)^{1/3}$. Substituting this constraint into (2.4) gives precisely (2.3) for $N = 5$. However, the physical interpretation of the effective superpotential is not the same as in the electric description, since the canonically normalized degrees of freedom are different. The position of the ground state in the confined theory is then determined by a balance in the superpotential (2.4) between the scale dependence of the gaugino condensate and the Dirac mass term. Parametrically, for $\lambda\Lambda_2 \ll \widehat{\Lambda}_5$, the field expectation values and vacuum energy scale as $\phi \sim (\Lambda_5^{13}/(\lambda^3\Lambda_2^4))^{1/9}$ and $V \sim |(\lambda^{12}(\Lambda_2/\Lambda_5)^{10})^{1/9}\Lambda_5^4|$. In order for this weakly coupled confined description to be the relevant one, the expectation values in the ground state must be much smaller than the confinement scale Λ_2. This requires $\Lambda_5 \ll \lambda^{3/13}\Lambda_2$, which is the opposite limit for applicability of the weak coupling electric description discussed in the previous subsection.

2.3. Dual Descriptions

For $N = 7$ and 9, in the limit $\Lambda_2 \gg \Lambda_N$, the $SU(N)$ is weakly gauged at the scale Λ_2 and may be treated as a weakly gauged subgroup of an $SU(N+1)_F$ flavor symmetry under which P and L are not distinguished. The $SU(2)$ theory therefore has $\frac{1}{2}(N+1)$ flavors ($N+1$ $\square$) and flows in the infrared towards an interacting fixed point in a non-Abelian Coulomb phase [1]. For $\Lambda_N \ll \lambda^{(N-2)/(3M-2)}\Lambda_2$ the $SU(2)$ is strongly coupled in the ground state and near the fixed point. In the region of the fixed point there are two dual descriptions of the interacting theory, either of which may be used to describe the low energy theory [1]. As discussed in the introduction, duality has only been conjectured for interacting theories precisely at a fixed point. However, if the dual descriptions make sense physically, they should apply in a neighborhood of the fixed point. One of the strongly coupled descriptions is in terms of the original electric fields and gauge group (2.1).

The dual magnetic description for $N = 7$ and 9 has gauge group $SU(N) \times \widetilde{SP}(\frac{1}{2}(N-5))$

with matter content

$$
\begin{array}{ccc}
 & SU(N) \times \widetilde{SP}(\frac{1}{2}(N-5)) & \\
\widehat{A} & (\begin{smallmatrix}\Box\\ \Box\end{smallmatrix}, 1) & \\
\widehat{F} & (\Box, 1) & \\
\widetilde{\overline{P}} & (\overline{\Box}, \Box) & \\
\widetilde{L} & (1, \Box) & \\
\overline{U} & (\overline{\Box}, 1) & \\
\overline{D} & (\overline{\Box}, 1). &
\end{array}
\tag{2.5}
$$

The dual descriptions are just the Affleck, Dine Seiberg $SU(N)$ theories with an $\begin{smallmatrix}\Box\\ \Box\end{smallmatrix}$ and $N-4\ \overline{\Box}$ [6], with the maximal $SP(\frac{1}{2}(N-5))$ flavor symmetry acting on the $\overline{\Box}$ promoted to a gauge symmetry, additional matter to cancel anomalies, and an extra flavor of $\Box$ and $\overline{\Box}$. The fields $\widehat{A} = A/\Lambda_2 = P^2/\Lambda_2$ and $\widehat{F} = F/\Lambda_2 = PL/\Lambda_2$ are confined $SU(2)$ "mesons" while $\widetilde{\overline{P}}$ and $\widetilde{L}$ are dual "magnetic" quarks (throughout tilded fields represent canonically normalized "magnetic" degrees of freedom). For expectation values much less than Λ_2, the fields (2.5) represent the canonically normalized degrees of freedom in the dual description. In the absence of the electric tree level superpotential (2.2), the moduli space of the dual theories are parameterized by $Z = A\overline{UD}$, $X_1 = F\overline{D}$, $X_2 = F\overline{U}$, $Y = A^{(N-1)/2}F$, $V = A\widetilde{\overline{P}}\widetilde{\overline{P}}$, and $R = F\widetilde{\overline{P}}\widetilde{L}$ subject to the dual tree level superpotential

$$
W_{\widetilde{tree}} = \frac{1}{\Lambda_2}\left(V + R\right). \tag{2.6}
$$

This superpotential, along with the non-perturbative dual gauge dynamics discussed below, ensure that the moduli space of the dual theory coincides with the classical moduli space of the electric theory [1]. Just as in the electric theory, the gauge group is generically broken to $SU(N-2) \subset SU(N)$ on the dual moduli space. It should be noted that because of D-term constraints, the dual quarks can not gain an expectation value on the moduli space.

The exact superpotential over the dual moduli space is given by

$$
W = 2\left(\frac{\Lambda_N^{3N-2}\Lambda_2^3}{YZV^{(N-5)/2}}\right)^{1/2} - \frac{N-3}{2}\left(\frac{Y}{\Lambda_2^{(11-N)/2}}\right)^{2/(N-3)} + W_{\widetilde{tree}} + \lambda X_1, \tag{2.7}
$$

The first term arises from gaugino condensation in an unbroken $SU(2) \subset SU(N-2) \subset SU(N)$. The terms $W_{\widetilde{tree}} = \widehat{A}\widetilde{\overline{P}}\widetilde{\overline{P}} + \widehat{F}\widetilde{\overline{P}}\widetilde{L}$ are Yukawa couplings in the dual theory. For $Y \neq 0$ the dual quarks gain a mass from these terms. Gaugino condensation in the dual $\widetilde{SP}(\frac{1}{2}(N-5))$ then gives rise to the second term. The final term is the magnetic

operator corresponding to the electric tree level superpotential (2.2), and gives a Dirac mass $m = \lambda\Lambda_2$ to the pair $\widetilde{F}$ and $\overline{D}$.

For $\lambda\Lambda_2 \gg \widehat{\Lambda}_5$, the Dirac pair $\widehat{F\overline{D}}$ may be integrated out of the dual description. The effective theory is then the Affleck, Dine, Seiberg $SU(N)$ model with a gauged flavor symmetry, subject to the tree level superpotential $W_{\widetilde{tree}} = \widehat{A\widetilde{P}\widetilde{P}}$. Since the Yukawa coupling and dual gauge coupling are large, the dual theory does not have a weak coupling limit. The vacuum energy presumably scales as $V \sim \widehat{\Lambda}_5^4$.

For $\lambda\Lambda_2 \ll \widehat{\Lambda}_5$, the Dirac pair $\widehat{F\overline{D}}$ are light compared to the dynamical scale and can not be integrated out. Because of the large dual tree level Yukawa coupling and strong dual gaugino condensation, the Y and V moduli may be integrated out. The effective superpotential in this limit is precisely (2.3). Again, the physical interpretation of (2.3) is not the same as in the electric description since the relevant degrees of freedom are different. The strong $\widetilde{SP}(\frac{1}{2}(N-5))$ gauge dynamics does not allow a quantitative estimate of the physical quantities in the ground state. In this limit the strong coupling would appear as large corrections to the Kahler potential for Z, X_i, and R.

2.4. Free Magnetic Description of Another Theory

For $N \geq 11$ the $SU(2)$ is not asymptotically free. In this case the theory becomes strongly coupled at short distances and the degrees of freedom (2.1) can not be the relevant ones in the far ultraviolet. The question then naturally arises as to whether the theory (2.1) for $N \geq 11$ can be interpreted as an effective weakly coupled magnetic description of another "electric" theory which *is* asymptotically free. A class of electric theories which are asymptotically free and have (2.1) as a free magnetic phase are

$$
\begin{array}{cc}
 & SU(N) \times SP(\frac{1}{2}(N-5)) \\
 & \\
A & (\begin{smallmatrix}\square\\\square\end{smallmatrix}, 1) \\
\overline{P} & (\overline{\square}, \square) \\
L & (1, \square) \\
\overline{U} & (\overline{\square}, 1)
\end{array}
\tag{2.8}
$$

for N odd. These theories have the same matter content as the dual theories in the previous subsection with the extra $SU(N)$ flavor removed.

In order to write the gauge invariants of this theory it is useful to define $V_{\alpha\beta} = A\overline{P}_\alpha \overline{P}_\beta$, and $Q_\alpha = A\overline{P}_\alpha \overline{U}$, where α, β are $SP(M)$ indices. The classical moduli space is then parameterized by V^k and $QV^{k-1}L$, $k = 1, \ldots, \frac{1}{2}(N-5)$, with the $SP(M)$ indices

contracted with the invariant antisymmetric matrix. On the moduli space the gauge group is generically broken to $SU(5) \subset SU(N)$, with $\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$ and $\overline{\Box}$ of $SU(5)$ remaining. At tree level there is a single renormalizable coupling which can be added to the superpotential

$$W_{tree} = \lambda V. \tag{2.9}$$

This superpotential leaves invariant a non-anomalous $U(1)_R$ symmetry, and completely lifts the classical moduli space. Classically there is a supersymmetric ground state at the origin.

Quantum mechanically, the non-perturbative $SU(N)$ dynamics lifts the classical supersymmetric ground state at the origin and supersymmetry is broken. The low energy description of supersymmetry breaking in the ground state depends on the relative importance of the $SU(N)$ and $SP(\frac{1}{2}(N-5))$ non-perturbative dynamics. If the $SP(M)$ is weakly coupled in the ground state, it may be treated classically. The position of the ground state is then determined by a balance between the potential generated by the unbroken $SU(5)$ with $\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$ and $\overline{\Box}$ and the tree level potential. For $\lambda \ll 1$ the expectation values along the moduli space, and vacuum energy scale as $\phi \sim \lambda^{-13/(4N+7)}\Lambda_{SU}$ and $V \sim \lambda^{(4N-20)/(2N+3)}\Lambda_{SU}^4$ [6]. In order for this approximation to be valid, the $SP(\frac{1}{2}(N-5))$ must be weakly coupled at the scale of the expectation values, which requires $\lambda^{-13/(4N+7)}\Lambda_{SU} \gg \Lambda_{SP}$.

If the $SP(\frac{1}{2}(N-5))$ is strongly coupled in the ground state, its non-perturbative dynamics can not be ignored. For $\Lambda_{SP} \gg \Lambda_{SU}$, $SU(N)$ is weakly gauged at the scale Λ_{SP}, and may be treated as a weakly gauged flavor symmetry. The $SP(\frac{1}{2}(N-5))$ therefore has $\frac{1}{2}(N+1)$ flavors ($N+1$ $\Box$) and for $N \geq 11$ flows in the infrared towards a weakly coupled theory in a free magnetic phase. The weakly coupled magnetic description has gauge group $SU(N) \times \widetilde{SU}(2)$ with "mesons" $\widehat{\overline{A}} = \overline{A}/\Lambda_{SP} = \overline{P}^2/\Lambda_{SP} \in \overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}$ of $SU(N)$ and $\widehat{\overline{D}} = \overline{D}/\Lambda_{SP} = \overline{P}L/\Lambda_{SP} \in \overline{\Box}$ of $SU(N)$, and dual "magnetic" quarks $\widetilde{P} \in (\Box,\Box)$ of $SU(N) \times \widetilde{SU}(2)$ and $\widetilde{L} \in \Box$ of $\widetilde{SU}(2)$. For expectation values much less than Λ_{SP} these fields, along with the electric fields A and $\overline{U}$ make up the canonically normalized degrees of freedom. The matter content of this free magnetic phase is just that of the $SU(N) \times SU(2)$ model (2.1) with an additional flavor of $\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$ and $\overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}$ of $SU(N)$. The scale $\widehat{\Lambda}_{SU}$ of the magnetic theory is related to Λ_{SU} by the matching condition $\widehat{\Lambda}_{SU}^{2N} = \Lambda_{SU}^{2N+3}/\Lambda_{SP}^3$ at the scale Λ_{SP}.

In the absence of the electric tree level superpotential (2.9), the moduli space of the free magnetic theory is parameterized by $Z = \widetilde{P}^2\overline{U}\overline{D}$, $X_1 = \widetilde{P}\widetilde{L}\widehat{\overline{D}}$, $X_2 = \widetilde{P}\widetilde{L}\overline{U}$, $\overline{V} = \widehat{\overline{A}}\widetilde{P}\widetilde{P}$,

and $V = \overline{A}A$, subject to the dual tree level superpotential

$$W_{\widetilde{tree}} = \frac{1}{\Lambda_{SP}}\left(\overline{V} + X_1\right). \tag{2.10}$$

This superpotential, along with the non-perturbative $\widetilde{SU}(2)$ dynamics ensures that the moduli space of the free magnetic theory coincides with classical moduli space of the electric theory. With the electric tree level superpotential (2.9), the full tree level superpotential in the magnetic theory is

$$W_{tree} = W_{\widetilde{tree}} + \lambda V. \tag{2.11}$$

It follows from symmetries, holomorphy, and limits that there are no additional contributions to the magnetic tree level superpotential. The final term is a Dirac mass $m = \lambda\Lambda_{SP}$ for the pair A and $\widehat{\overline{A}}$. For $\lambda\Lambda_{SP} \gg \Lambda_{SU}$ the Dirac pair is much heavier than the dynamical scale in the free magnetic theory and may be integrated out. Below the scale $\lambda\Lambda_{SU}$, the effective magnetic theory is then given by (2.1) with superpotential (2.3) and matching condition $\widehat{\Lambda}_N^{3N-2} = (\lambda\Lambda_{SP})^{N-2}\widehat{\Lambda}_{SU}^{2N} = \lambda^{N-2}\Lambda_{SP}^{N-5}\Lambda_{SU}^{2N+3}$. In this limit we therefore see that the free magnetic description of (2.8) is precisely the $SU(N)\times SU(2)$ theory (2.1) for $N \geq 11$. Note that the Yukawa coupling in the effective theory, $W = \widetilde{P}\widetilde{L}\widehat{\overline{D}}$, is not small, so in this limit the $SU(N)$ is not weakly coupled in the ground state. The vacuum energy presumably scales as $V \sim \widehat{\Lambda}_N^4$.

3. Generalizations

There are many generalizations of the applications of duality and confinement to supersymmetry breaking introduced here and in [4]. A direct generalization of the $SU(N)\times SU(2)$ models are $SU(N)\times SP(M)$ models discussed in the Appendix. Generalizations to other product gauge groups with similar matter content are straightforward.

Most known models of dynamical supersymmetry breaking can be obtained from $SU(N)$ with $A \in \begin{smallmatrix}\Box\\ \Box\end{smallmatrix}$ and $N-4$ fields $\in \overline{\Box}$ by reducing the $SU(N)$ to product gauge groups [10]. This can be accomplished in a full theory with the addition of vector like matter to spontaneously break the original gauge group to a product. The addition of vector matter can not affect supersymmetry breaking in the full theory. Consider the case of adding an adjoint, Φ, with general renormalizable superpotential

$$W = m\Phi^2 + g\Phi^3. \tag{3.1}$$

For large m, this theory has, in addition to the original $SU(N)$ theory at $\Phi = 0$, a variety of disconnected vacua with low energy effective theories given by

$$
\begin{array}{lcl}
 & SU(n_1) \times SU(n_2) \times U(1) & \\
 & & \\
A_1 & (\begin{smallmatrix}\square\\\square\end{smallmatrix}, 1)_{2n_2} & \\
A_2 & (1, \begin{smallmatrix}\square\\\square\end{smallmatrix})_{-2n_1} & \\
P & (\square, \square)_{n_2 - n_1} & \\
\overline{Q}_i & (\overline{\square}, 1)_{-n_2} & i = 1 \ldots n_1 + n_2 - 4 \\
\overline{Q}'_i & (1, \overline{\square})_{n_1} & i = 1 \ldots n_1 + n_2 - 4,
\end{array}
\tag{3.2}
$$

with $N = n_1 + n_2$ odd. Due to the massless charged matter, the $U(1)$ has no interesting dynamics. The classical $U(1)$ D-terms are however crucial in some instances in lifting certain directions in field space [10,11]. Starting from (3.2) with $n_1 = 3$, $n_2 = 2$ results in the $SU(3) \times SU(2) \times U(1)$ model. For some models it is possible to simply ignore the $U(1)$ and remove some matter from (3.2) to obtain an effective theory which breaks supersymmetry, although this is not gauranteed [10]. Starting with $n_2 = 2$, ignoring the $U(1)$, removing both A_1 and A_2, and $(n_1 + n_2 - 6)$ $\overline{Q}_i$, and $(n_1 + n_2 - 5)$ $\overline{Q}'_i$ gives the $SU(N) \times SU(2)$ models. Ignoring the $U(1)$, removing A_1 and A_2, and $(n_1 - 4)$ $\overline{Q}_i$, and $(n_2 - 4)$ $\overline{Q}'_i$, gives $SU(N) \times SU(M)$ models [12].

As another possibility consider adding an flavor of Ω and $\overline{\Omega} \in \begin{smallmatrix}\square\\\square\end{smallmatrix}$ and $\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$ with the (non-renormalizable) superpotential

$$
W = m\Omega\overline{\Omega} + g(\Omega\overline{\Omega})^2. \tag{3.3}
$$

For large m, this theory yields a variety of disconnected vacua with low energy theories

$$
\begin{array}{lcl}
 & SU(n_0) \times SP(n_1) & \\
 & & \\
A_0 & (\begin{smallmatrix}\square\\\square\end{smallmatrix}, 1) & \\
A_1 & (1, \begin{smallmatrix}\square\\\square\end{smallmatrix}) & \\
P & (\square, \square) & \\
\overline{Q}_i & (\overline{\square}, 1) & i = 1 \ldots n_0 + 2n_1 - 4 \\
L_i & (1, \square) & i = 1 \ldots n_0 + 2n_1 - 4,
\end{array}
\tag{3.4}
$$

with $N = n_0 + 2n_1$. The field A_1 can be given a mass by adding a term $W = A^2\widetilde{\Omega}^2$ to the superpotential. These theories are similar to the ones in sections (2.3) and (2.4). Removing A_0 and $(n_0 - 4)$ $\overline{Q}_i$, and $(n_0 + 2n_1 - 5)$ L_i gives the $SU(N) \times SP(M)$ models of the Appendix. Application of confinement or duality to any models obtained from (3.2) or (3.4) is also straightforward.

4. Conclusions

Identifying the relevant low energy degrees of freedom in models of supersymmetry breaking is important in giving a proper description of the ground state. The simplest example is for weakly coupled theories with a single dynamical scale which break supersymmetry by a dynamically generated superpotential. The ground state sits near the classical moduli space and the relevant degrees of freedom are the classical moduli fields. In addition, the relevant non-perturbative superpotential in this limit is the exact superpotential over the classical moduli space. As demonstrated here, if the ground state sits at strong coupling near the origin of moduli space, additional light non-perturbative degrees of freedom can be relevant to the low energy description of supersymmetry breaking. In fact for theories with multiple dynamical scales it is quite likely that some of the low energy degrees of freedom are confined or magnetic.

If the strong dynamics is confining or in a free magnetic phase a weak coupling description of the supersymmetry breaking in terms of different gauge groups and matter content than the electric description can be obtained in this regime. By adjusting parameters of the model the confined or free magnetic description can often be continously connected to a weakly coupled electric description. Supersymmetry breaking can be explicitly verified in both limits. It is important to note that since supersymmetry is broken, there can in principle be phase transitions as a function of the parameters of the model. So the existence of two weakly coupled descriptions does not gaurantee that supersymmetry is broken for all values of the parameters. However, the confinement or duality in this case does act as a generator for another weakly coupled model of supersymmetry breaking.

If the strong dynamics is in a non-Abelian Coulomb phase near an interacting fixed point the dual descriptions formally do not have a weak coupling limit. In this case there is more than one interacting description of the supersymmetry breaking. By adjusting the parameters of the theory to move the ground state far enough from the fixed point, one of the duals is more weakly coupled and provides the most relevant description.

Confinement and duality in supersymmetry breaking models can also have important phenomenological consequences for parameters of the low energy theory. A confined or magnetic description can have mass scales which appear unnatural in the low energy theory, as for the mass of Dirac pair in the confined or dual descriptions of the $SU(N) \times SU(2)$ models. Alternately, a small dimensionless parameter can appear in a renormalizable low energy theory if the underlying electric theory is non-renormalizable.

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Appendix A. $SU(N) \times SP(M)$ Generalizations

The $SU(N) \times SU(2)$ models of section 2 may be generalized to $SU(N) \times SP(M)$ models with matter content

$$\begin{array}{lll} & SU(N) \times SP(M) & \\ & & \\ P & (\square, \square) & \\ L & (1, \square) & \\ \overline{Q}_i & (\overline{\square}, 1) & i = 1 \ldots 2M, \end{array} \tag{A.1}$$

with N odd. For any N and M, a superpotential is generated by one (and only one) of the two gauge groups: for $M \leq \frac{1}{2}(N-1)$ it is generated by $SU(N)$ dynamics and for $M \geq \frac{1}{2}(N+1)$ it is generated by $SP(M)$ dynamics. The electric version of the models with $M \leq \frac{1}{2}(N-1)$ were discussed in Ref. [10]. The quantum modification of the moduli space for $M = \frac{1}{2}(N-1)$ and quantum removal of flat directions for $M = \frac{1}{2}(N+1)$ were discussed in Ref. [5]. The models with $M \leq \frac{1}{2}(N-3)$ can have confining or dual descriptions analogous to the $SU(N) \times SU(2)$ models of section 2.

For $M \leq \frac{1}{2}(N-1)$ the classical moduli space is parameterized by $Z_{ij} = P^2 \overline{Q}_{[i} \overline{Q}_{j]}$ and $X_i = PL\overline{Q}_i$ with the gauge group generically broken to $SU(N-2M) \subset SU(N)$. The tree level superpotential

$$W_{tree} = \lambda X_1 + \sum_{i,j>2} \gamma^{ij} Z_{ij}. \tag{A.2}$$

completely lifts the moduli space [10,5]. Classically there is a supersymmetric ground state at the origin. Quantum mechanically the non-perturbative $SU(N)$ dynamics lifts the classical supersymmetric ground state and supersymmetry is broken. As with the $SU(N) \times SU(2)$ models, the relevant description of the supersymmetry breaking ground state depends on the relative importance of the $SU(N)$ and $SP(M)$ non-perturbative dynamics.

In order to have a weakly coupled regime below the scale of the non-renormalizable operators which appear in (A.2), we assume that $\gamma^{ij} \ll \Lambda_{SU}^{-1}$ and Λ_{SP}^{-1}. The Z_{ij} $i,j \geq 2$ moduli are lifted only the non-renormalizable terms in (A.2) and therefore have a much smaller classical potential than the other moduli in this limit. As a consequence, for $\lambda \ll 1$, the quantum mechanical ground state develops large expectation values along these directions. The resulting low energy theories are precisely the $SU(N) \times SU(2)$ models of section 2, with $N \to N+2-2M$, along with the light singlets Z_{ij}, $i,j \geq 2$, with the superpotential (A.2). If the $SP(M)$ is weakly coupled at the scale of the expectation values, this is the relevant description of the ground state. In the following we briefly mention aspects of the limits in which the $SP(M)$ dynamics is important.

For $2M+3 \leq N < 6M+5$ the $SP(M)$ theory is asymptotically free and can be dualized as in [1,13] to the theory

$$\begin{array}{lll} & SU(N) \times SP(\widetilde{M}) & \\ A & (\begin{smallmatrix}\Box\\ \Box\end{smallmatrix}, 1) & \\ F & (\Box, 1) & \\ \overline{P} & (\overline{\Box}, \Box) & \\ \widetilde{L} & (1, \Box) & \\ \overline{Q}_i & (\overline{\Box}, 1) & i = 1 \dots 2M, \end{array} \tag{A.3}$$

with $\widetilde{M} \equiv \frac{1}{2}(N-2M-3)$, and tree level superpotential

$$W_{\widetilde{tree}} = A\overline{P}\overline{P} + F\overline{P}\widetilde{L} + \lambda\Lambda_{SP}F\overline{Q}_1 + \sum_{i,j>2} \gamma^{ij}\Lambda_{SP}A\overline{Q}_i\overline{Q}_j. \tag{A.4}$$

As in section 2, $A = P^2/\Lambda_{SP}$ and $F = PL/\Lambda_{SP}$. Note that the non-renormalizable term in the tree level superpotential (A.2) has become renormalizable in the dual theory. For $\gamma\Lambda_{SP} \ll 1$, $A\overline{Q}_i\overline{Q}_j$ get large expectation values, leading to (2.5) with $N \to N+2-2M$ as the low energy theory.

For $2M = N-3$, the $SP(\widetilde{M})$ in (A.3) is trivial, revealing that $SP(M)$ confines. The confined theory (A.3) in this case was discussed as an electric theory in [14]. The relation between (A.1) and (A.3) for $2M = N-3$ was also noted in [15], where it was generalized to include additional $SU(N)$ fundamental flavors. For $2M+3 < N \leq 3M+2$, the $SP(\widetilde{M})$ dual in (A.3) is not asymptotically free and is therefore free in the infrared; this range does not occur for the $M=1$ theories discussed in section 2. For $3M+2 < N < 6M+5$, both the original theory (2.8) and its dual (A.3) flow to an interacting fixed point without a supersymmetric vacuum.

For $N \geq 6M+5$, (A.1) is not asymptotically free but can be interpreted as the low energy description of the asymptotically free theory

$$
\begin{array}{lcc}
 & SU(N) \times SP(\widetilde{M}) & \\
A & (\begin{smallmatrix}\Box\\ \Box\end{smallmatrix}, 1) & \\
\overline{P} & (\overline{\Box}, \Box) & \\
\widetilde{L} & (1, \Box) & \\
\overline{Q}_i & (\overline{\Box}, 1) & i = 1 \ldots 2M,
\end{array}
\tag{A.5}
$$

with $\widetilde{M} = \frac{1}{2}(N-3-2M)$ and the general, renormalizable, tree level superpotential

$$
W_{tree} = \widetilde{\lambda} A \overline{PP} + \sum_{i,j} \widetilde{\gamma}^{ij} A \overline{Q}_i \overline{Q}_j . \tag{A.6}
$$

For $\widetilde{\gamma} \ll 1$, $A\overline{Q}_i\overline{Q}_j$ get large expectation values, leading to (2.8) with $N \to N+2-2M$ as the low energy theory. Dualizing $SP(\widetilde{M})$, (A.5) leads to (A.1) with $\overline{Q}_1 = \overline{P}^2 \widetilde{L}/\Lambda_{\widetilde{SP}}$ and additional fields $A \in \begin{smallmatrix}\Box\\ \Box\end{smallmatrix}$ and $\overline{A} = \overline{P}^2/\Lambda_{\widetilde{SP}} \in \overline{\begin{smallmatrix}\Box\\ \Box\end{smallmatrix}}$ of $SU(N)$, and a superpotential

$$
W_{tree} = \overline{A} P^2 + \overline{Q}_1 P L + \widetilde{\lambda} \Lambda_{SP} A \overline{A} + \sum_{i,j} \widetilde{\gamma}^{ij} A \overline{Q}_i \overline{Q}_j . \tag{A.7}
$$

Integrating out the massive $A\overline{A}$ pair results in the theory (A.1) with the tree level superpotential (A.2) with $\lambda = 1$ and $\gamma = -\widetilde{\gamma}(\widetilde{\lambda}\Lambda_{SP})^{-1}$. The non-renormalizable theories discussed in the beginning of this appendix are thus obtained as the low energy description of renormalizable theories.

References

[1] N. Seiberg, hep-th/9411149, Nucl. Phys. B 435 (1995) 129.

[2] N. Seiberg, hep-th/9402044, Phys. Rev. D 49 (1994) 6857.

[3] K. Intriligator and N. Seiberg, hep-th/9509066.

[4] K. Intriligator and S. Thomas, talk presented at *Unification: From the Weak Scale to Planck Scale*, Institute for Theoretical Physics, Santa Barbara, CA, Oct. 23, 1995; Massachusetts Institute of Technology, Oct. 1995.

[5] K. Intriligator and S. Thomas, SLAC-PUB-7041, hep-th/9603158, to appear in Nucl. Phys. B.

[6] I. Affleck, M. Dine, and N. Seiberg, Nucl. Phys. B 256 (1985) 557.

[7] I. Affleck, M. Dine, and N. Seiberg, Phys. Lett. B 137 (1984) 187

[8] D. Amati, K. Konishi, Y. Meurice, G. Rossi, and G. Veneziano, Phys. Rep. 162 (1988) 169 and references therein.

[9] H. Murayama, hep-ph/9505082, Phys. Lett. B 355 (1995) 187.

[10] M. Dine, A. Nelson, Y. Nir, and Y. Shirman, hep-ph/9507378, Phys. Rev. D 53 (1996) 2658.

[11] C. Csaki, L. Randall, and W. Skiba, MIT-CTP-2532, hep-th/9605108; C. Csaki, L. Randall, W. Skiba, and R. Leigh, MIT-CTP-2543, hep-th/9607021.

[12] E. Poppitz, Y. Shadmi, S.P. Trivedi, hep-th/9605113.

[13] K. Intriligator and P. Pouliot, hep-th/9505006, Phys. Lett. B 353 (1995) 471.

[14] E. Poppitz and S. Trivedi, hep-th/9507169, Phys. Lett. B 365 (125) 1996.

[15] P. Pouliot, hep-th/9510148, Nucl. Phys. B 367 (151) 1996.